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A.I.M.D., Fairness and Fractal Scaling of TCP Traffic

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Abstract: We propose a natural and simple model for the joint throughput evolution of a set of TCP sessions sharing a common bottleneck router, via products of random matrices. Surprisingly enough, even when based on the assumption of i.i.d losses, this model is sufficient to explain the fractal scaling of aggregates of TCP traffic at short time scales, which was recently identified by a statistical analysis based on wavelets by Feldmann, Gilbert, Willinger et al. in [8, 7]. This model has various other implications, also described in the paper, such as the quantification of instantaneous fairness, the characterization of the autocorrelation function of inter-loss periods, or the evaluation of the performance degradation due to synchronization of losses in the shared bottleneck router.

Key-words: TCP, additive increase–multiplicative decrease algorithm, wavelet, fractal, IP traffic, product of random matrices, Pareto distribution, synchronisation, fairness, autocorrelation.

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A.A.D.M., Equité et Propriétés Fractales du Trafic TCP

Résumé : Nous proposons un modèle élémentaire, à base de produits de matrices aléatoires, pour l'évolution jointe des débits d'un ensemble de sources TCP partageant un routeur commun considéré comme goulot d'étranglement. Même dans le cas de pertes i.i.d., ce modèle suffit à expliquer le caractère fractal du trafic agrégé, aux petites échelles de temps, caractère récemment mis en évidence par des analyses statistiques utilisant des ondelettes par Feldmann, Gilbert, Willinger et al. dans [8, 7]. Nous décrivons plusieurs autres applications de ce modèle, comme la quantification de l'inéquité instantanée du protocole, la caractérisation de l'autocorrélation du processus des inter-pertes, ou encore l'évaluation de la dégradation des performances due à une synchronisation des pertes dans le routeur partagé.

Mots-clés : TCP, algorithme des accroissements additifs et de la décroissance multiplicative, ondelette, fractale, trafic IP, produit de matrices aléatoires, loi de Pareto, synchronisation, équité, autocorrélation.

1 Introduction

The present paper proposes a simple model for the joint evolution of TCP sessions sharing a common bottleneck router.

Throughout the paper, we will refer to this model, which is based on mere products of random matrices, as to the *AIMD* (additive increase, multiplicative decrease) model. In this model, which is described in §2, TCP is not represented at packet level, but rather via simple fluid equations that describe the joint evolutions of throughput for the set of sessions sharing the router, and the loss process in the router. The validity of this fluid vision of the problem is of course debatable. In §2.2, we discuss the simplifying assumptions that are used to allow one to reduce the actual behavior of the set of sessions to the AIMD model.

Section 3 focuses on the characterization of the steady state of the AIMD model (mean values, covariances, autocorrelation functions), which is the basis of the analysis in the rest of the paper. As we will see, even under homogeneous assumptions, where the long terms averages of throughput are the same for all sessions (which translates the so called fairness of TCP), the stationary solution of the AIMD model is such that there is a heavy tailed dispersion of throughputs for different sessions at any given time. By this, we mean that under the fluid approximation of the AIMD model, the ratio of the fastest to the slowest session at any given time is a random variable with a Pareto type tail. In §3.3, we introduce the notion of *dispersion ratio* and that of *instantaneous fairness*, and show that what is usually called fairness (and which should be called fairness in mean) may be compatible with heavy instantaneous unfairness in the case of moderately synchronized i.i.d. losses. The analysis also leads to the characterization of the autocorrelation function of inter-loss periods, which is a useful information in relation with such models as that of Altman et al. [2], where this data is assumed to be given.

Section 4 probably contains the most surprising practical result obtained from the AIMD model. It focuses on a potential explanation for the short time scale fractal properties of TCP traffic which were recently identified using wavelet statistical tools by Feldmann, Gilbert, Willinger et al. in [8, 7]. By short time scales, we mean the same as in the above papers. In particular, this concerns time scales where the effect of the superposition of a large number of on-off sessions with Pareto on periods cannot be invoked to explain a non trivial scaling. In our model, there are no heavy tails in the primitive data at all: the number of active sessions *does not vary* and sessions are *saturated* (i.e. there are no on or off periods with heavy tails) and losses are assumed to be i.i.d. It is of course possible to add such variations to our model, and also to replace the fluid model by a packet level model taking slow start into account; however, this will not be done in the present paper for the following reason: all this can only increase variability and reinforce the fractal properties observed in our simplified model. In other words, the main conclusions on high variability and fractal scaling should hold true (at least on a qualitative basis) for such refined models since they hold true for the less variable basic AIMD model.

The connections between this fractal scaling property and instantaneous unfairness are also discussed.

Section 5 focuses on another natural application of the AIMD model namely the evaluation of the performance degradation due to synchronization of losses in the shared bottleneck router.

It is beyond the scope of the present paper to review in detail the connections with the vast literature on the relationship between losses and TCP throughput (see e.g. [12], [13], [2]). However, the two following general comments should be made:

- the AIMD model focuses on the interaction between a large number of sources, whereas the formulas relating throughput to losses in these papers concern a single session;
- in contrast to what is considered in these papers, the basic data for the AIMD model is the probability that a source experiences a loss at a congestion epoch (see π and p below), rather than the intensity of the losses experienced by a single source.

2 The AIMD Model

In this section, we propose a set of fluid evolution equations allowing one to represent the key features of the AIMD mechanism for N homogeneous TCP sessions sharing one bottleneck router. We will first consider the homogeneous case, where all sessions have the same RTT. We then generalize the approach to the heterogeneous case with different RTT's.

By definition, the n-th *congestion time* is the n-th epoch at which a loss or several simultaneous losses occur on this shared router. We use the following notation:

- N is the number of TCP sessions, which we assume to be constant with time;
- C is the capacity of the bottleneck router;
- $X_n^{(i)}$ is the throughput of session i just after the n-th congestion time;
- $W_n^{(i)}$ is the window size of session i just after the n-th congestion time;
- T_n is n-th congestion time;
- $\tau_{n+1} = T_{n+1} - T_n$ is the time between the n-th and the $n+1$ st congestion times;
- $R^{(i)}$ is the mean RTT of session i (in the homogeneous situation, $R^{(i)} = R$);
- $a_n^{(i)}$ is a 0,1 valued random variable with value 1 if session i experiences a loss at the n-th congestion time, 0 otherwise;
- $p^{(i)}$ is the loss probability for session i: $p^{(i)} = \mathbb{E}a_n^{(i)}$ (in the homogeneous situation, $p^{(i)} = p$);
- $\alpha^{(i)}$ is the linear growth rate of the window size of session i with time; it makes sense to take $\alpha^{(i)} = 1/R^{(i)}$;
- β is the multiplicative reduction of window size in case of loss (1/2 by default).

We will only consider the case of independent losses. By this, we mean the following:

1. the random variables $\{a_n^{(i)}, i = 1, \ldots, N\}$ are independent in n;

2. for all n, the random variables $a_n^{(i)}, i = 1, \ldots, N$ are generated independently, with $\mathbb{P}(a_0^{(i)} = 1) = \pi^{(i)}$, but only the samples such that at least one of them is 1 are kept (by definition, there is at least one loss at a congestion epoch). The resulting conditional law is easy to compute. For instance, in the homogeneous case,

$$\mathbb{P}(a_0^{(i)} = 1) \quad = \quad p = \frac{\pi}{1-(1-\pi)^N}$$

and for $i \neq j$,

$$\begin{aligned}
\mathbb{P}(a_0^{(i)} = 1, a_0^{(j)} = 1) &= \frac{\pi^2}{1-(1-\pi)^N} \\
\mathbb{P}(a_0^{(i)} = 0, a_0^{(j)} = 1) &= \frac{\pi(1-\pi)}{1-(1-\pi)^N} \\
\mathbb{P}(a_0^{(i)} = 0, a_0^{(j)} = 0) &= \frac{(1-\pi)^2(1-(1-\pi)^{N-2}}{1-(1-\pi)^N}.
\end{aligned}$$

2.1 Homogeneous case

In our model, we assume that the throughput and the window size are linked by a Little like law:

$$W_n^{(i)} = X_n^{(i)} R, \tag{1}$$

which is a first simplifying assumption (this way of linking throughput to window is heuristic; Little's law would only apply to stationary means whereas we use it for linking instantaneous values here). Then the evolution of the throughput is given by:

$$X_{n+1}^{(i)} = ((1 - a_{n+1}^{(i)}) + \beta a_{n+1}^{(i)}) \left(X_n^{(i)} + \frac{\alpha}{R} \tau_{n+1} \right). \tag{2}$$

We now use the fact that losses occur as soon as the router capacity is reached to get the following relation between the $X_n^{(i)}$'s and τ_{n+1}:

$$\sum_{i=1}^{N} X_n^{(i)} + \frac{\alpha}{R} \tau_{n+1} N = C. \tag{3}$$

At this stage we assume that the buffer capacity of the router is 0 or negligible (see §2.2 for a simple way to relax this assumption).

So when using the notation

$$\gamma_n^{(i)} = (1 - a_n^{(i)}) + \beta a_n^{(i)},$$

we get

$$\begin{aligned}
X_{n+1}^{(i)} &= \gamma_{n+1}^{(i)} \left(X_n^{(i)} + \frac{C}{N} - \frac{1}{N} \sum_{j=1}^{N} X_n^{(j)} \right) \\
&= \gamma_{n+1}^{(i)} \left(\frac{C}{N} + (1 - \frac{1}{N}) X_n^{(i)} - \frac{1}{N} \sum_{j \neq i} X_n^{(j)} \right).
\end{aligned} \tag{4}$$

Let $Z_n^{(i)} = X_n^{(i)}/\rho$ with $\rho = 1/N$. Let B_n be the N-dimensional random vector with coordinates $\gamma_n^{(i)} C$ and let A_n be the $N \times N$ random matrix

$$\begin{pmatrix} \gamma_n^{(1)}(1-\rho) & -\gamma_n^{(1)}\rho & \cdots & \cdots & -\gamma_n^{(1)}\rho \\ -\gamma_n^{(2)}\rho & \gamma_n^{(2)}(1-\rho) & -\gamma_n^{(2)}\rho & \cdots & -\gamma_n^{(2)}\rho \\ \vdots & & \ddots & & \vdots \\ -\gamma_n^{(N-1)}\rho & \cdots & -\gamma_n^{(N-1)}\rho & \gamma_n^{(N-1)}(1-\rho) & -\gamma_n^{(N-1)}\rho \\ -\gamma_n^{(N)}\rho & \cdots & \cdots & -\gamma_n^{(N)}\rho & \gamma_n^{(N)}(1-\rho) \end{pmatrix}. \tag{5}$$

We have

$$Z_{n+1} = A_{n+1} Z_n + B_{n+1}. \tag{6}$$

2.2 Some preliminary remarks on the model

A natural concern is the relationship between the behavior of the X_n process in the neighborhood of the boundaries of its state space, i.e. when $\sum_i X_n^{(i)} \sim C$ or $X_n^{(i)} \sim 0$ for some i, where the fluid and the discrete models seem to differ significantly at first glance.

- **Buffer Capacity** In the AIMD model, when $\sum_i X_n^{(i)}$ reaches C, losses occur. This is of course unrealistic in that the buffer size should be taken into account. Let B be the buffer size of the shared router. We show that the above equations can be adapted to take buffering into account, and that the adaptation simply consists in replacing C by $C_n = C + x_n$, with $x_n \leq \sqrt{2BN}/R$. The justification is the following: if at time 0 the total arrival rate is C and the buffered fluid amount is 0, then at time t the total arrival rate is $C + Nt/R^2$ and the buffer size $Nt^2/2R^2$. Thus, when the buffer size B is reached, the total arrival rate is equal to $C + \sqrt{2BN}/R$. At the time the total arrival rate reaches C, if the buffered fluid amount is larger than 0, then we get $x_n < \sqrt{2BN}/R$. Remark that the total arrival rate just after the n-th congestion time cannot be larger than C (otherwise the buffered fluid amount cannot decrease and losses will take place until the total arrival rate is below C).

- **Small Throughputs and Timeouts** Windows cannot be halved an unbounded number of times, be it only because they always remain larger than or equal to 1, whereas, in our model, one specific coordinate of X can actually be halved an unbounded number of times. We argue that although windows cannot be halved infinitely often indeed, throughputs (which are the actual state variables of the AIMD model) can very well become "multiplicatively small" in case of repeated timeouts. In such a case the doubling of the RTO variable (see e.g. [16]) has the very same qualitative effect on session inter-packet times and therefore throughput as the one of the AIMD model. So in spite of the fact that timeouts are apparently not present in the AIMD model, one can argue that their presence is actually taken into account via the behavior of the model in the neighborhood of the corresponding boundary.

- **Large Throughputs** Throughputs are often limited by the presence of a maximal window size; this will not be taken into account here.

It is clear that such a fluid model cannot take into account fine packet level aspects of TCP. This is intrinsic; models that mix this high level fluid evolution of the window sizes and packet level as e.g. in [3] can of course be contemplated, but seem out of reach for the time being. Nevertheless we believe that the AIMD model captures key phenomena present in the protocol, and its behavior close to the boundary of the state space is certainly not a very accurate representation of reality, but is qualitatively correct.

2.3 Non homogeneous case

The generalization to the heterogeneous case gives the following equations:

- Little's law (in the heuristic sense)

$$W_n^{(i)} = X_n^{(i)} R^{(i)}. \tag{7}$$

- Throughput evolution

$$X_{n+1}^{(i)} = \gamma_{n+1}^{(i)} \left(X_n^{(i)} + \frac{\alpha^{(i)}}{R^{(i)}} \tau_{n+1} \right). \tag{8}$$

- Relation between $X_n^{(i)}$'s and τ_{n+1}

$$\sum_{i=1}^{N} X_n^{(i)} + \tau_{n+1} \sum_{j=1}^{N} \frac{\alpha^{(i)}}{R^{(j)}} = C. \tag{9}$$

- The throughputs dynamics are then given by

$$X_{n+1}^{(i)} = \gamma_{n+1}^{(i)} \left(\frac{\frac{\alpha^{(i)}}{R^{(i)}}}{\sum_j \frac{\alpha^{(j)}}{R^{(j)}}} C + \left(1 - \frac{\frac{\alpha^{(i)}}{R^{(i)}}}{\sum_j \frac{\alpha^{(j)}}{R^{(j)}}} \right) X_n^{(i)} - \frac{\frac{\alpha^{(i)}}{R^{(i)}}}{\sum_j \frac{\alpha^{(j)}}{R^{(j)}}} \sum_{j \neq i} X_n^{(j)} \right). \tag{10}$$

Let $Z_n^{(i)} = X_n^{(i)} / \rho^{(i)}$ with $\rho^{(i)} = \frac{\frac{\alpha^{(i)}}{R^{(i)}}}{\sum_j \frac{\alpha^{(j)}}{R^{(j)}}}$. This leads to:

$$Z_{n+1}^{(i)} = \gamma_{n+1}^{(i)} \left(C + (1 - \rho^{(i)}) Z_n^{(i)} - \sum_{j \neq i} \rho^{(j)} Z_n^{(j)} \right).$$

Let B_n be the random vector with coordinates $\gamma_n^{(i)} C$ and let A_n be the $N \times N$ random matrix

$$\begin{pmatrix}
\gamma_n^{(1)}(1-\rho^{(1)}) & -\gamma_n^{(1)}\rho^{(2)} & \cdots & \cdots & -\gamma_n^{(1)}\rho^{(N)} \\
-\gamma_n^{(2)}\rho^{(1)} & \gamma_n^{(2)}(1-\rho^{(2)}) & -\gamma_n^{(2)}\rho^{(3)} & \cdots & -\gamma_n^{(2)}\rho^{(N)} \\
\vdots & & \ddots & & \vdots \\
-\gamma_n^{(N-1)}\rho^{(1)} & \cdots & \cdots & \gamma_n^{(N-1)}(1-\rho^{(N-1)}) & -\gamma_n^{(N-1)}\rho^{(N)} \\
-\gamma_n^{(N)}\rho^{(1)} & \cdots & \cdots & -\gamma_n^{(N)}\rho^{(N-1)} & \gamma_n^{(N)}(1-\rho^{(N)})
\end{pmatrix}. \tag{11}$$

We again have

$$Z_{n+1} = A_{n+1} Z_n + B_{n+1}. \tag{12}$$

3 Steady State Solution

We are interested in the steady state solutions of (6) and (12). For certain mean values, there is no need of detailed analysis. For instance, consider the homogeneous case, and assume that there exists a stationary regime Z such that $\mathbb{E}(Z^{(i)})$ is finite and the same for all i. Since each line of A_n sums up to 0, when taking expectation on both sides of (6), we get $\mathbb{E}[Z^{(i)}] = \mathbb{E}[B^{(i)}]$, for all i, that is

$$\mathbb{E}[X^{(i)}] = \frac{C}{N}\left(1-(1-\beta)p\right).$$

The term $(1-\beta)p$ measures the loss of performance due to the idleness of the router. We will evaluate this more precisely in §5. In particular, if $\beta = 1/2$, then for all i,

$$\mathbb{E}[X^{(i)}] = \frac{C}{N}\left(1-\frac{p}{2}\right). \tag{13}$$

Immediate calculations then give the following expression for the mean stationary inter congestion time:

$$\mathbb{E}[\tau] = \frac{CR^2 p}{2N} = \frac{CR^2\pi}{2N(1-(1-\pi)^N)}, \tag{14}$$

at least when taking $\alpha = 1/R$, which will be assumed to hold in what follows.

Further characteristics require more sophisticated tools.

Theorem 1 *Under the above assumptions, there exists a unique finite stationary solution* $\{\mathcal{Z}_n\}$, $n \in \mathbb{Z}$, *to (12), with*

$$\mathcal{Z}_n = B_n + A_n B_{n-1} + A_n A_{n-1} B_{n-2} + A_n A_{n-1} A_{n-2} B_{n-3} + \cdots, \quad n \in \mathbb{Z}, \tag{15}$$

where we have used the fact that the sequences of i.i.d. matrices and vectors A_n *and* B_n *could be continued to negative values of* n. *The unique stationary regime* $\mathcal{X}_n$ *of the throughput is obtained from* $\mathcal{Z}_n$ *via the formula*

$$\mathcal{X}_n^{(i)} = \rho^{(i)} \mathcal{Z}_n^{(i)}. \tag{16}$$

In the homogeneous case,

$$\mathcal{Z}_n^{(i)} = N \mathcal{X}_n^{(i)}. \tag{17}$$

Proof From results of [4] (see also [5] for a survey on the matter), the only thing to verify is that the corresponding series are convergent. That the series defining $\mathcal{Z}_n$ is convergent, follows from taking expectations: using the independence assumption, we get that the expectation of the R.H.S. of (15) is equal to

$$\mathbb{E}(B_1) + \sum_{k\geq 1} \left(\mathbb{E}(A_1)\right)^k \mathbb{E}(B_1) = \sum_{k \geq 0} \left(\mathbb{E}(A_1)\right)^k \mathbb{E}(B_1).$$

Here

$$\mathbb{E}(A_1) = \mathbb{G}(\mathbb{I} - \mathbb{R}),$$

where $\mathbb{G}$ is the $N \times N$ diagonal matrix with entries $\mathbb{G}_{ii} = \mathbb{E}\gamma_1^{(i)} = 1-(1-\beta)\mathbb{E}(a_1^{(i)})$, $\mathbb{I}$ is the $N \times N$ identity matrix and $\mathbb{R}$ is the $N \times N$ square matrix with elements $\mathbb{R}_{ij} = \rho^{(j)}$ for all i, j.

So $\mathbb{E}(A_1) \leq \mathbb{G}$, and $\mathbb{G}$ is a diagonal matrix with all its entries positive and strictly less than 1, which proves that the series of the $\mathbb{E}(A_1)^k$ is bounded from above by a convergent series.

From the fact that $\mathbb{R}$ is idempotent ($\mathbb{R}^2 = \mathbb{R}$), we have $(\mathbb{I}-\mathbb{R})^k = (\mathbb{I}-\mathbb{R})$ for all $k \geq 1$, and from the bound $\mathbb{E}(A_1) \geq -g\mathbb{R}$, where $0 < g = \max_i \mathbb{G}_{ii} < 1$, we also get that series of the $\mathbb{E}(A_1)^k$ is bounded from below by a convergent series.

This concludes the proof of the convergence of the expectations, which in turn implies the almost sure convergence of the random series defined in (15). □

3.1 Covariance Matrix

For all random vectors X, we will denote $\mathbb{C}(X)$ its covariance matrix:

$$\mathbb{C}(X) = \mathbb{E}\left((X-\mathbb{E}X)(X-\mathbb{E}X)^t\right), \tag{18}$$

where X^t is the transpose of X.

Theorem 2 *In the homogeneous case, the covariance matrix of $\mathcal{Z} = \mathcal{Z}_0$ is given by the following formula:*

$$\mathbb{C}(\mathcal{Z}) = C^2\left(\left(\frac{a-b}{1-a+\rho(a-b)}\right)\mathbb{I} + \left((b-c) - \frac{b\rho(a-b)}{1-a+\rho(a-b)}\right)\mathbb{J}\right), \tag{19}$$

where $\mathbb{J}$ is the matrix with all its entries equal to 1 and

$$a = \mathbb{E}[(\gamma_0^{(1)})^2] \quad = \quad \frac{(1-3\pi/4)-(1-\pi)^N}{1-(1-\pi)^N} \tag{20}$$

$$b = \mathbb{E}[\gamma_0^{(1)}\gamma_0^{(2)}] \quad = \quad \frac{(1-\pi/2)^2-(1-\pi)^N}{1-(1-\pi)^N} \tag{21}$$

$$c = \overline{\gamma}^2 = \mathbb{E}[\gamma^{(1)}]^2 \quad = \quad \left(1-\frac{\pi/2}{1-(1-\pi)^N}\right)^2. \tag{22}$$

When N grows large and $\pi < 1$,

$$\mathbb{C}(\mathcal{Z}) \sim C^2\left(\frac{1-\pi}{3}\mathbb{I} - \frac{(1-\pi/2)^2}{3N}\mathbb{J}\right). \tag{23}$$

Proof We have

$$\mathbb{C}(\mathcal{Z}) = \mathbb{C}\left(\sum_{k\geq 0} A_0 \cdots A_{1-k}B_{-k}\right).$$

Note that

$$\begin{aligned}\mathbb{C}\left(\sum_{k\geq 0} A_0 \cdots A_{1-k}B_{-k}\right) \quad &= \quad \sum_{k\geq 0}\sum_{l\geq 0}\mathbb{E}\left(A_0A_{-1}\cdots A_{1-k}B_{-k}B_{-l}^tA_{1-l}^t\cdots A_{-1}^tA_0^t\right)\\ &\quad -\mathbb{E}\left(A_0A_{-1}\cdots A_{1-k}B_{-k}\right)\mathbb{E}\left(B_{-l}^tA_{1-l}^t\cdots A_{-1}^tA_0^t\right).\end{aligned}$$

We will use the following notation (some of which are already used in the proof of Theorem 1):

- Γ_n is the diagonal matrix with entries $(\Gamma_n)_{ii} = \gamma_n^{(i)}$;
- $\mathbb{E}(\Gamma_n) = \mathbb{G}$, the diagonal matrix with entries $\mathbb{E}\gamma_0^{(1)} = \sqrt{c}$;
- $\mathbb{R} = \rho\mathbb{J}$, with $\rho = 1/N$;

Notice that we have: $A_n = \Gamma_n(\mathbb{I} - \mathbb{R})$, and that $(\mathbb{I} - \mathbb{R})\mathbb{J}$ is the null matrix. Using this, we obtain the following expressions:

$$\begin{aligned} \mathbb{E}B_0B_0^t &= C^2\left((a-b)\mathbb{I} + b\mathbb{J}\right) \\ \mathbb{E}B_0\mathbb{E}B_0^t &= C^2c\mathbb{J}, \end{aligned}$$

so that

$$\mathbb{E}B_0B_0^t - \mathbb{E}B_0\mathbb{E}B_0^t = C^2\left((a-b)\mathbb{I} + (b-c)\mathbb{J}\right).$$

Similarly,

$$\begin{aligned} \mathbb{E}A_0B_{-1}B_{-1}^tA_0^t &= C^2\mathbb{E}\left(\Gamma_0(\mathbb{I}-\mathbb{R})((a-b)\mathbb{I}+b\mathbb{J})(\mathbb{I}-\mathbb{R})^t\Gamma_0^t\right) \\ &= C^2\mathbb{E}\left(\Gamma_0(\mathbb{I}-\mathbb{R})((a-b)\mathbb{I})(\mathbb{I}-\mathbb{R})^t\Gamma_0^t\right) \\ &= C^2(a-b)\mathbb{E}\left(\Gamma_0(\mathbb{I}-\mathbb{R})\Gamma_0^t\right) \\ &= C^2(a-b)(a-\rho(a-b))(\mathbb{I}-\mathbb{R}), \end{aligned}$$

where we used the relations $\mathbb{E}(\Gamma_0\Gamma_0^t) = (a-b)\mathbb{I} + b\mathbb{J}$ and

$$\mathbb{E}(\Gamma_0\mathbb{R}\Gamma_0^t) = (a - \rho(a-b))\mathbb{I} - b\mathbb{R}.$$

Since in addition,

$$\mathbb{E}A_0B_{-1}\mathbb{E}B_{-1}^tA_0^t = 0,$$

we get

$$\mathbb{E}A_0B_{-1}B_{-1}^tA_0^t - \mathbb{E}A_0B_{-1}\mathbb{E}B_{-1}^tA_0^t = C^2((a-\rho(a-b))\mathbb{I} - b\mathbb{R}).$$

More generally, one finds by the same argument that for all $k \geq 1$,

$$\mathbb{E}A_0A_{-1}\cdots A_{1-k}B_{-k}B_{-k}^tA_{1-k}^t\cdots A_{-1}^tA_0^t = C^2(a-b)((a-\rho(a-b))\mathbb{I} - b\mathbb{R})^{k-1}((a-\rho(a-b))\mathbb{I} - b\mathbb{R}),$$

whereas

$$\mathbb{E}A_0A_{-1}\cdots A_{-k+1}B_{-k}\mathbb{E}B_{-k}^tA_{-k+1}^t\cdots A_{-1}^tA_0^t = 0,$$

and that for all $l \neq k$,

$$\mathbb{E}A_0A_{-1}\cdots A_{1-l}B_{-l}B_{-k}^tA_{1-k}^t\cdots A_{-1}^tA_0^t = 0$$

and

$$\mathbb{E}A_0A_{-1}\cdots A_{-k+1}B_{-k}\mathbb{E}B_{-l}^tA_{-l+1}^t\cdots A_{-1}^tA_0^t = 0.$$

Hence the result for (19). The last relation is obtained from the following asymptotics:

$$\begin{aligned} a &= 1 - 3\pi/4 + o(1); \\ b &= (1-\pi/2)^2 + o(1); \\ a - b &= \frac{\pi(1-\pi)}{4} + o(1); \\ b - c &= o(N^{-1}). \end{aligned}$$

$\square$

Remarks Notice that $a - c = \mathrm{var}(\gamma_0^{(i)})$ and that $b - c = \mathrm{cov}(\gamma_0^{(i)}, \gamma_0^{(j)})$, $i \neq j$. Simple calculations show that $b - c < 0$. It is not so surprising to find that two different coordinates of $\mathcal{Z}$ are negatively correlated as the corresponding sessions compete for capacity.

We can rephrase the results of Theorem 2 as follows: in the homogeneous case,

$$\mathrm{var}(\mathcal{X}^{(i)}) = \frac{C^2}{N^2}\left(\frac{(a-b)(1-\rho b) + (a-b)(1-a+\rho(a-b))}{1-a+\rho(a-b)}\right); \tag{24}$$

$$\mathrm{cov}(\mathcal{X}^{(i)}, \mathcal{X}^{(j)}) = \frac{C^2}{N^2}\left((b-c) - \frac{b\rho(a-b)}{1-a+\rho(a-b)}\right). \tag{25}$$

From (3) and Theorem 2, we immediately get the following expression for the variance of the stationary inter-loss times:

$$\begin{aligned} \mathrm{var}(\tau) &= \frac{R^4}{N^2}\mathrm{var}(\sum_i \mathcal{X}^{(i)}) \\ &= \frac{R^4}{N^2}(N\mathrm{var}(\mathcal{X}^{(1)} + N(N-1)\mathrm{cov}(\mathcal{X}^{(1)}, \mathcal{X}^{(2)})). \end{aligned} \tag{26}$$

3.2 Autocorrelation

For all random vectors X, Y, we will denote $\mathbb{C}(X,Y)$ the covariance matrix:

$$\mathbb{C}(X,Y) = \mathbb{E}\left((X - \mathbb{E}X)(Y - \mathbb{E}Y)^t\right). \tag{27}$$

Using techniques similar to the ones of Theorem 2, one easily gets the following theorem.

Theorem 3 *In the homogeneous case, the autocorrelation matrices of the $\mathcal{Z}_n$ vectors are given by the following formula:*

$$\mathbb{C}(\mathcal{Z}_0, \mathcal{Z}_k) = \overline{\gamma}^k\left(\mathbb{C}(\mathcal{Z}) - C^2(a-c)\mathbb{R}\right). \tag{28}$$

As a direct corollary, we obtain the following formula for the autocorrelation of the inter loss time process:

$$\mathrm{cov}(\tau_0, \tau_k) = \frac{R^4}{N}\overline{\gamma}^k\left(\mathrm{var}(\mathcal{X}^{(1)}) + (N-1)\mathrm{cov}(\mathcal{X}^{(1)}, \mathcal{X}^{(2)}) - (a-c)\right). \tag{29}$$

This formula comes as a natural complement to the formulas given in [2], which allow one compute continuous time throughput ergodic averages for TCP from the knowledge of the inter loss time autocorrelation function.

3.3 Instantaneous Fairness

Roughly speaking, the *instantaneous unfairness* of TCP should be a measure of the dispersion of the stationary throughputs at a given time, in the stationary regime. This dispersion is completely characterized by the joint distribution of the coordinates of $\mathcal{Z}_1$ defined in (15). Let us see how to capture this notion in more compact ways.

Let us first concentrate on the homogeneous case. For H a fixed subset of $\{1, \dots, N\}$, we denote

$$T^H = \sum_{i \in H} \mathcal{X}^{(i)},$$

the aggregate of stationary throughputs over set H. We define the *dispersion ratio* for aggregates of size j, where $1 \le j \le N$, as the random variable

$$d_j = \frac{\max_{H,|H|=j} T^H}{\min_{H,|H|=j} T^H}.$$

Theorem 4 *For all integers j, for x large,*

$$\mathbb{P}(d_j > x) \ge D' x^{-\xi}, \tag{30}$$

with D' a positive constant and

$$\xi = \frac{\ln(\mu)}{\ln(\frac{j+N}{2N})}, \quad \mu = \frac{\pi^j (1-\pi)^{N-j}}{1-(1-\pi)^N}. \tag{31}$$

Proof Let $I_n = \{0 \le i \le N : \gamma_n^{(i)} = 1/2\}$ and

$$S(n) = \sum_{i=1}^{N} \mathcal{X}_n^{(i)}, \quad S_-(n) = \sum_{i \in I_n} \mathcal{X}_n^{(i)}, \quad S_+(n) = \sum_{i \notin I_n} \mathcal{X}_n^{(i)}.$$

With this notation, $\tau_{n+1} = \frac{C-S(n)}{N} R^2$ and $C = S_+(n) + 2S_-(n) = S(n) + S_-(n)$, so that from (4),

$$\mathcal{X}_{n+1}^{(i)} = \gamma_n^{(i)} \left(\mathcal{X}_n^{(i)} + \frac{S_-(n)}{N} \right).$$

If $I_n = I_{n+1} = I$, this in turn implies

$$S_-(n+1) \quad = \quad \frac{N+|I|}{2N} S_-(n). \tag{32}$$

So, if $I_n = I_{n+1} = \cdots = I_{n+K} = I$, for some $k > 0$, then

$$\begin{aligned} S_-(n+k) &= \left(\frac{N+|I|}{2N} \right)^k S_-(n) \\ &\le \left(\frac{N+|I|}{2N} \right)^k C. \end{aligned} \tag{33}$$

So, for all $\epsilon > 0$, if $k = k(\epsilon)$ denotes the smallest integer such that $\left(\frac{N+|I|}{2N}\right)^k C \le \epsilon$, then

$$\mathbb{P}(S_-(n+k) \le \epsilon) \ge \mathbb{P}(I_n = I_{n+1} = \cdots = I_{n+k} = I) = \mu^{k+1},$$

where μ denotes the probability that $I_n = I$, conditional on the fact that there is at least one loss, which is given in (31). Direct calculations based on logarithms lead to the equivalence

$$\mu^k \sim D \epsilon^{\xi},$$

with D a constant and ξ defined in (31).

Similarly, if J is a fixed subset of $\{1,\ldots,N\}$, disjoint of I and such that $|I| = |J|$ (we assume that $|I|$ is small enough for this to be possible), then

$$T^J(n+1) \quad = \quad T^J(n) + \frac{|I|}{N} S_-(n). \tag{34}$$

So, if $I_n = I_{n+1} = \cdots = I_{n+K} = I$, for some $k > 0$, with $|I| = j$, then

$$T^J(n+k) = T^J(n) + \frac{j}{N} \frac{1 - \left(\frac{j+N}{2N}\right)^{k+1}}{1 - \frac{j+N}{2N}} S_-(n) \geq S_-(n) \frac{2j}{N-j} \left(1 - \left(\frac{j+N}{2N}\right)^{k+1}\right). \tag{35}$$

Let now $k = k(\epsilon)$ denote the smallest integer such that

$$\left(\frac{j+N}{2N}\right)^k \leq \epsilon \frac{2j}{N-j} \left(1 - \left(\frac{j+N}{2N}\right)^{k+1}\right),$$

where ϵ is small. Then

$$\begin{aligned} \mathbb{P}\left(\frac{\max_{H,|H|=j} T^H(n+k)}{\min_{H,|H|=j} T^H(n+k)} \geq \frac{1}{\epsilon}\right) \quad &\geq \quad \mathbb{P}\left(I_n = \cdots = I_{n+k} = I, \frac{T^J(n+k)}{T^I(n+k)} \geq \frac{1}{\epsilon}\right) \\ &= \quad \mathbb{P}(I_n = I_{n+1} = \cdots = I_{n+k} = I) \\ &= \quad \mu^{k+1} \\ &\sim \quad D' \left(\frac{1}{\epsilon}\right)^{-\xi}, \end{aligned} \tag{36}$$

with D' a constant and ξ defined as in (31) □

So the dispersion ratio has a heavy tail of the Pareto type. We conclude from this that one may very well have at the same time *fairness in expectation*, in that in steady state, all sessions have the same mean throughput, and nevertheless a severe *instantaneous unfairness* in that at any given time, the ratio of best throughput to worst is actually a heavy tailed random variable.

The log-log complementary plot of the distribution function of the dispersion ratio d_1 (which is also the ratio of max throughput over min) as obtained from simulations of the AIMD model, is given in Figure 1. Here $N = 100$ and $p = 0.01$. This ratio is invariant w.r.t. the values of C and R. The slope of the decreasing part of this curve is compatible with the lower bound estimate of (31).

In order to check that this effect is not simply an artefact stemming from the model, we have used NS to evaluate the dispersion ratio under the more accurate packet level description of TCP offered by NS. Figure 2 gives the log-log complementary plot of the distribution function of the dispersion ratio d_1 as obtained via the NS simulation of 100 TCP flows sharing a 10 000 packets/s router. The router is FIFO and with a buffer of 70 packets; the minimal value of RTT is .1 seconds. One should not try to compare this curve with that of Figure 1 since the two simulated systems have no direct relations (in general, it is not possible to impose a given proportion of losses at congestion epochs via NS). Note however that there is also a linear part over several scales, similar to that of the AIMD model case. We conclude from this that this heavy tailed dispersion is also present in NS simulations.

In the heterogeneous case, similar results can be obtained along the same lines. Of course throughput dispersion increases even more whenever RTT's themselves have more dispersion.

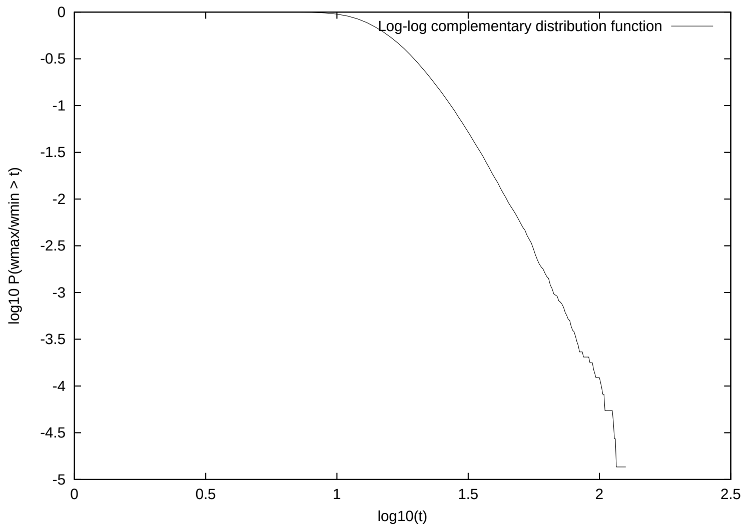

Figure 1: Log-log complementary distribution function of $\mathcal{X}_{\max}/\mathcal{X}_{\min}$ in the stationary AIMD model.

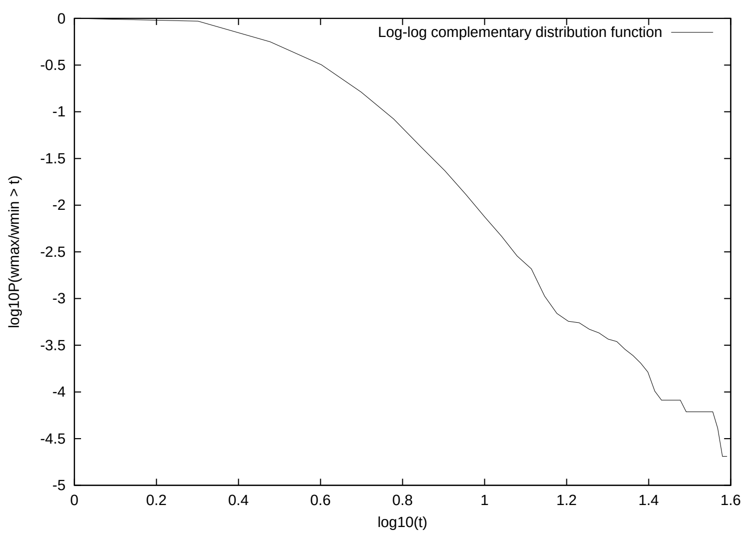

Figure 2: Log-log complementary distribution function of $\mathcal{X}_{\max}/\mathcal{X}_{\min}$ via NS.

So the heavy tailed dispersion of throughputs which is already present in the homogeneous case is reinforced in the inhomogeneous case.

Note that the proof of the heavy tailedness of dispersion extends directly to throughput-dependent loss probabilities (by this, we mean that the loss probability π of a source is a function of the current throughput of this source), provided these functions are bounded away from 0.

We would like to stress that the presence of heavy tailed distributions for the stationary distribution of certain affine dynamical systems based on random matrices with light tails (such as (6) and (12)) is not new. The first results along these lines are due to Kesten [11]. A survey on the matter can also be found in [5])

4 Fractal Scaling

In [10], the fractal scaling of certain real TCP traces was studied. The goal of this section is to show that stationary traces generated by the AIMD model already exhibit a mono-fractal scaling.

The section starts with simulation results based on the AIMD model and wavelet analysis on these simulations, similar to that used by Feldmann, Gilbert, Willinger and al. on real traces. We show that three key properties identified on real traces are also present in the AIMD traces:

1. Time aggregation leads to trajectories with important fluctuations over several scales;
2. The wavelet energy function plot yields a linear region starting at the lowest scales (high frequencies), and with a finite upper cutoff;
3. The wavelet partition function exhibits linearity characteristic of a mono fractal.

All these observations are compatible with a fractal behavior of the trajectories of the AIMD process (see e.g. [1], p. xxii). Some of these methods used require Gaussian assumptions (see e.g. [1]), This seems justified in the following framework where we focus on aggregates of K sessions (typically we take $K = \phi N$ sessions with $0 < \phi < 1$ and N large).

4.1 Some Simulations

4.1.1 Direct observation of scalability on AIMD traces

Figure 3 shows throughput traces generated from the AIMD model ($N = 100$, $p = 0.01$). We observe the sum of the throughputs of $Obs = 10$ sessions on 4 different time scales: the 4 first curves are generated from the sum of $Obs = 10$ session throughputs sampled at congestion times, namely at $\{T_n\}_{n=1..30000}$ (the value of the curve at point $x = n$ of the x axis is the value of this sum at T_n) and by aggregation of these over 10, 1000 and 1000 time units respectively; the 4 last curves display the evolution in continuous time for the number of packets sent by $Obs = 10$ sessions during $0.01, 0.1, 1, 10$ s.

This is to be compared with Figure 5 of [15]. Fluctuations remain quite important on each of these time scales. This could be linked to the results of Theorem 4 as important instantaneous dispersion of throughputs leads to stochastic phenomena over several time scales: think of the superposition of a random mixture of processes each with a given time scales but where the mixture is heavy tailed: aggregation does not lead to smoothness as long as there are still processes with higher time scale (lower frequencies).

4.1.2 Wavelet analysis

Given a time series $\{X_{n,k}, k = 1, \ldots, 2^n\}$, its discrete (Haar) wavelets coefficients $d_{n,k}$ are constructed as in [1],[7]. The average energy at scale j is then defined as:

$$E_j = \frac{1}{2^j} \sum_{k=1}^{2^j} |d_{j,k}|^2.$$

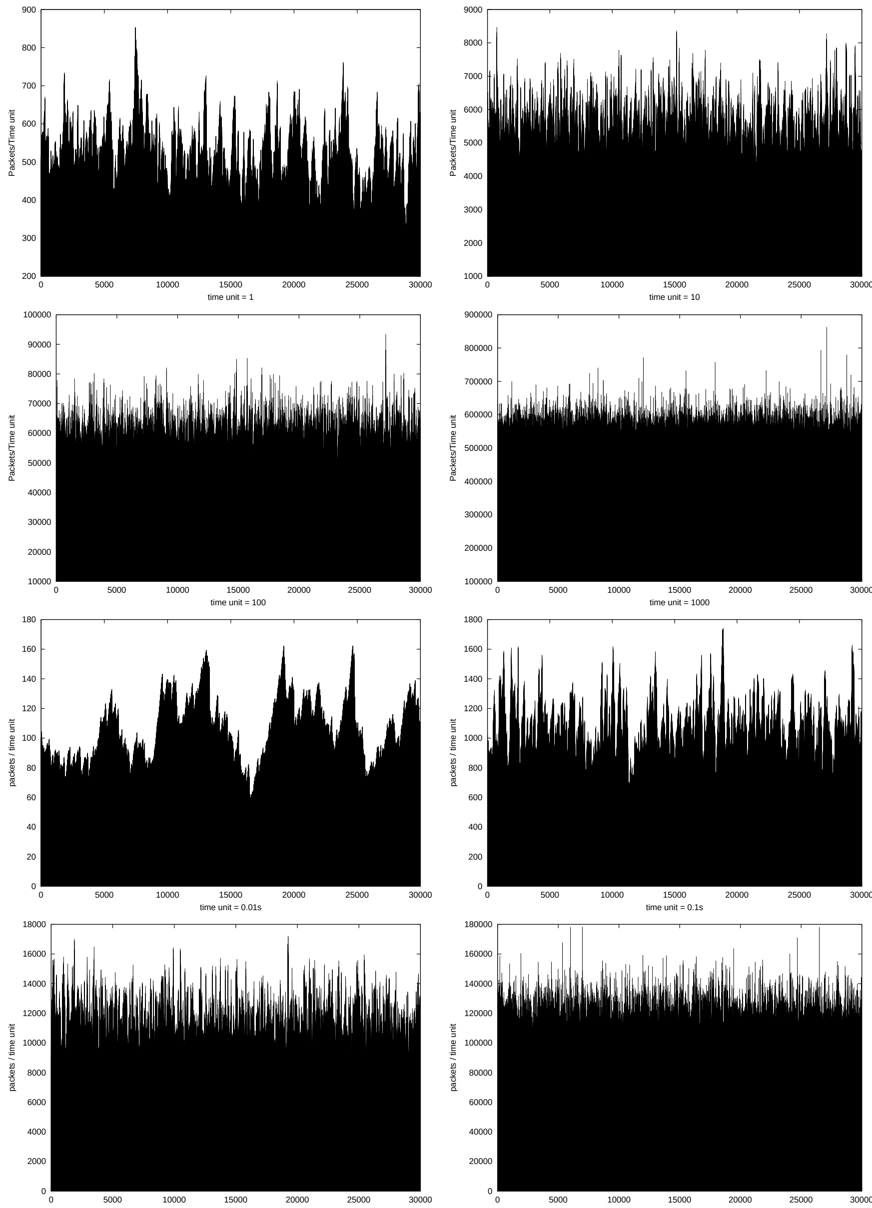

Figure 3: Traces generated from the AIMD model.

In Figure 4 we plot the evolution of the sum of the throughputs of $Obs = 10$, 100, 450 sessions in the case $N = 500$, $p = 0.1$ (again sampled at congestion times).
Figure 5 gives the logscale diagram (logarithm of the energy function as a function of the scale) of the traces of Figure 4 as defined in e.g. [1]: the 3 first diagrams correspond to a sampling at congestion times (Figure 4, $Obs = 10$, 100, 450); the last 3 curves display the evolution for the total number of packets sent by 10, 100 and 450 sessions during 0.1s as a function of time.

These plots are compatible with a (mono) fractal behavior in view of the fact that the scaling is concentrated at the lowest scales (highest frequencies) with an upper cutoff (here around the scales 7-8, the slope varies from 1.8 to 0.9.).

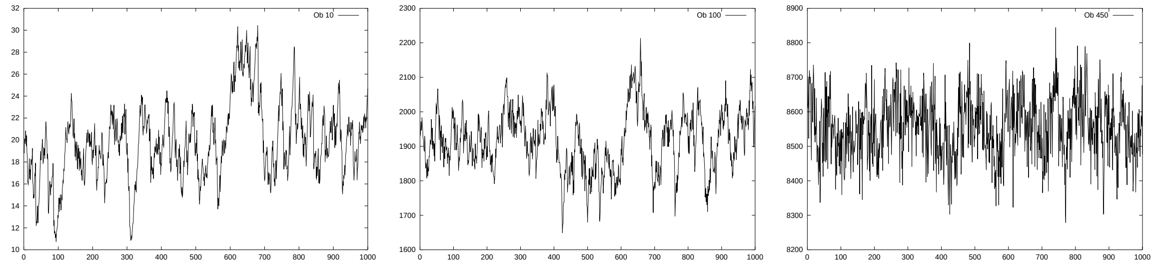

Figure 4: Plot of TCP simulation by AIMD.

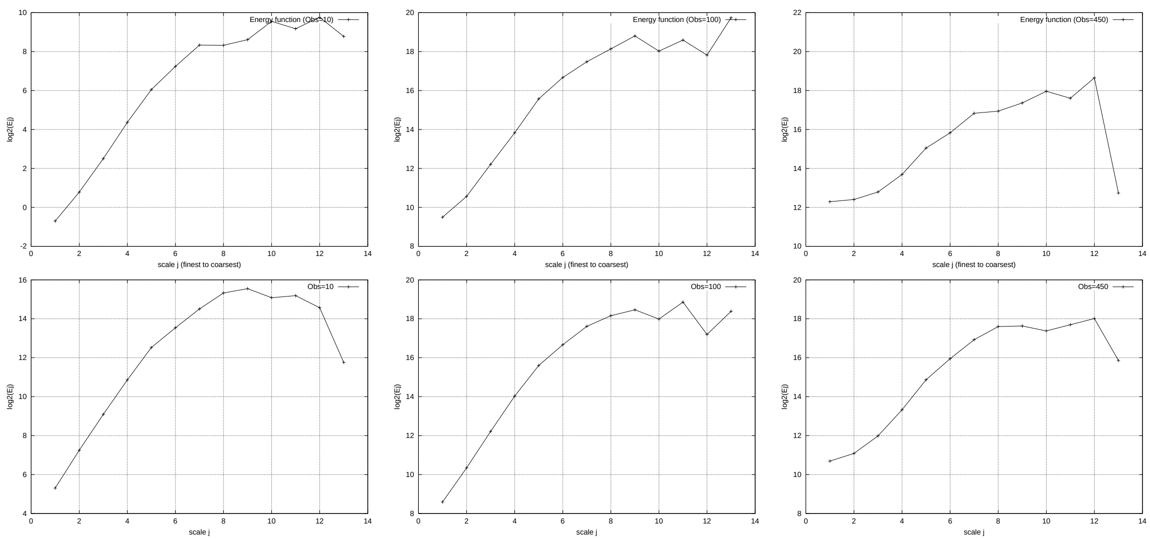

Figure 5: Plot of energy functions: TCP simulation by AIMD.

Generally speaking, the slope α of the linear part is related to the local regularity parameter h of the fractal. If $\alpha > 1$, in the Gaussian case, the sample paths are continuous and non-differentiable, whereas $\alpha < 1$ corresponds to non continuous paths [6] (in fact the steepest the slope, the more regular the paths as well illustrated by the two sets of figures).

In cases $Obs \leq 300$, we have $\alpha > 1$; then the local regularity parameter h of the fractal is given by the formula

$$h = \frac{\alpha - 1}{2}. \tag{37}$$

For instance, for $Obs = 10$, $\alpha \sim 2$ so that $h \sim 1/2$; whereas for $Obs = 300$, $\alpha \sim 1.3$ and $h = 0.15$.

On case $Obs = 450$, we have $\alpha \sim 0.9$; in this and other cases where $\alpha < 1$, sample paths are discontinuous.

The partition function at scale j is defined as:

$$S(q,j) = \sum_{k=1}^{2^j} |2^{j/2} d_{j,k}|^q.$$

In Figure 6, we plot the log of this function of the scale for $q = 0, 2, \ldots, 18$ ($N = 500$, $p = 0.1$, $Obs = 10$).

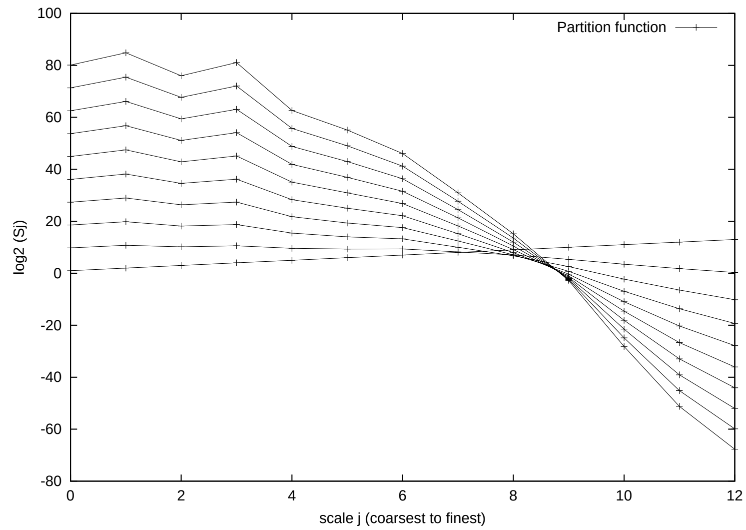

Figure 6: Plot of partition functions: TCP simulation by AIMD.

The linearity of the log-log plot of the partition function is also characteristic of a mono fractal [8].

Consistency with the value of the autocorrelation function Under stationary Gaussian assumptions, the regularity factor h of a stochastic process $\mathcal{T}(t)$ is related to its increments by the formula:

$$\mathbb{E}[(\mathcal{T}(t+s) - \mathcal{T}(t))^2] \sim |s|^{2h}, \tag{38}$$

when s tends to 0. Let $\mathcal{T}_n = \mathcal{T}_n^H$ be the H-aggregate of the stationary throughputs for $card(H) = K = \phi N$, at the n-th congestion time. When the mean inter loss time is small, one can use (38) to write the following rough heuristic expressions

$$\begin{aligned} \mathbb{E}[(\mathcal{T}_1 - \mathcal{T}_0)^2] &\sim (\mathbb{E}[\tau])^{2h} \\ \mathbb{E}[(\mathcal{T}_2 - \mathcal{T}_0)^2] &\sim (2\mathbb{E}[\tau])^{2h}, \end{aligned}$$

which in turn implies

$$h \sim \frac{1}{\ln(4)} \ln\left(\frac{\mathbb{E}[(\mathcal{T}_2 - \mathcal{T}_0)^2]}{\mathbb{E}[(\mathcal{T}_1 - \mathcal{T}_0)^2]}\right). \tag{39}$$

Using the results of Theorem 3 and the relation

$$\mathbb{E}[(\mathcal{T}(t+s) - \mathcal{T}(t))^2] = 2\left(\mathrm{var}(\mathcal{T}(t) - \mathrm{cov}(\mathcal{T}(t+s), \mathcal{T}(t))\right),$$

we can now compute the RHS which gives the following heuristic estimate of h when N is large:

$$h \sim \frac{1}{\ln(4)} \ln\left(\frac{1/3(1-\pi/4)(1-\pi+\phi) + (1-\pi/2)^2(1-\pi)\phi/4}{1/6(1-\pi/4)(1-\pi+\phi) + (1-\pi/2)^2(1-\pi)\phi/4}\right). \tag{40}$$

This estimate is consistent with the wavelet based estimates. For instance, if we take $\phi = 0.2$ and $p = 0.01$, both the wavelet estimate and (40) give $h \sim 0.4$.

Monfractals, Multifractals The traces studied by Feldmann, Gilbert, Willinger et al. revealed a multifractal scaling. So why do we find a monofractal here? Our guess is that the absence, in our model, of variations of the number of active sessions and of successions of on and off periods, could naturally explain a more steady behavior than that observed from real traces. The fact that we limit ourselves to a single bottleneck router could be another possible explanation.

5 Performance loss by synchronization

We will say that losses are synchronized when $a_n^{(i)} = 1$ for all n and i. We will say that losses are p-synchronized, with p a real close to 1 but less than 1, when $a_n^{(i)} = 1$ with probability p for all n and i.

Assume first that losses are synchronized, and all RTT's are the same. At T_n, all sessions halve their window. Let $S(t)$ denote the sum of the send rates of all sessions at time t. Let B be the buffer size of the shared router. Let $Q(t)$ be the total number of customers in the buffer at time t. In this model with non-empty buffer, we have (cf. §2.2)

$$S(T_n^-) = C + x \leq C + \frac{\sqrt{2BN}}{R}$$

where $0 \leq x < C$ (if $x \geq C$, the total send rate is always larger than C leading to full utilization of capacity).
In view of the AI rule, while $S(t) \leq C$,

$$S(T_n + t) = \frac{C+x}{2} + \frac{\alpha N}{R} t = \frac{C+x}{2} + \frac{N}{R^2} t$$

and

$$Q(T_n + t) = \left(B + \int_0^t (\frac{-C+x}{2} + \frac{N}{R^2} v) dv \right)^+ = \left(B + \frac{-C+x}{2} t + \frac{N}{2R^2} t^2 \right)^+ .$$

The minimum of the function

$$t \to B + \frac{-C+x}{2} t + \frac{N}{2R^2} t^2$$

is reached for

$$t^* = \frac{(C-x)R^2}{2N},$$

and the valued of the function at this minimum is

$$B - \frac{(C-x)^2 R^2}{4N} + \frac{(C-x)^2 R^2}{8N} = B - \frac{(C-x)^2 R^2}{8N}.$$

So we have under-utilization of the link capacity iff

$$B < \frac{(C-x)^2 R^2}{8N}$$

or equivalently

$$\frac{3\sqrt{2BN}}{CR} < 1.$$

If this condition is satisfied, the under-utilization can be evaluated as one minus the ratio between the total number of packets sent and the maximum number of packets that can be sent during a period $T = T_n - T_{n-1}$ which is here constant. Let us call this ratio $\mathbb{U}$. If ${}^*t$ is the first time at which $Q(T_n + t) = 0$, we have

$$ {}^*t = t^* - \frac{R^2}{N}\sqrt{\frac{(C-x)^2}{4} - \frac{2BN}{R^2}}. $$

Then

$$ \mathbb{U} = \frac{1}{CT}\left(C(t^* -^* t) - \int_{^*t}^{t^*} (\frac{C+x}{2} + \frac{Nu}{R^2})du \right). $$

After simplification, we have:

$$ \mathbb{U} = 0.25\left(1 - \frac{3\sqrt{2BN}}{CR}\right). $$

In case when we have deterministic p-synchronization, that is when at T_n there are exactly pN flows loosing packets and at T_{n+1} another group of pN flows loosing packets etc. In this case, calculations lead to the condition of under-utilization

$$ \frac{3\sqrt{2BN}}{pCR} < 1 $$

and

$$ \mathbb{U} = \frac{p}{3+p}\left(1 - \frac{3\sqrt{2BN}}{pCR}\right). $$

6 Conclusion and Future Work

We have proposed a natural model, called the AIMD model, allowing one to describe the evolution of a large number of TCP sessions sharing a common bottleneck router. We have shown that this model, which is based on mere random matrix products, captures the key features of AIMD and a qualitatively correct fluid approximation of TCP.

The model leads to closed form expressions for the steady state covariances (or the autocorrelation functions) of various quantities of interest such as the inter-loss times or the aggregated throughputs. This was exploited in several directions.

We first studied the case of moderate losses. A mathematical characterization of the instantaneous unfairness of the protocol was derived, and it was shown that even in the case where TCP is supposed to be fair, the dispersion ratio of throughputs is in fact heavy tailed. We also used wavelet tools similar to those of Feldmann, Gilbert, Willinger et al. to show that what is interpreted as a fractal scaling at short time scales on real TCP traffic traces is already present in the AIMD model. This seems to be linked to the high dispersion of throughputs.

Finally, we used this model to characterize what happens when losses are synchronized and what degradation may be expected due to some under utilization of the router's capacity in this case.

The basic model can be enriched in several ways. A first natural aim would be to have a finer interplay between such high level models of competition of several TCP sessions and detailed one-session packet level models such as the one proposed in [3]. Note that in such a refined model, the additional aggregation of packets due to slow start, and the additional variability due to the packet scale phenomena, can apparently only reinforce the dispersion and the local irregularity already present in the fluid AIMD model.

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